Lecture 7. ***Stability "in small", "in big" and "in general"***

In linear systems motion or balance, which are stable in small, may occur instable in the case of big deviations. Then the first Lyapunov's method is not enough for complete investigation of stability in nonlinear systems.

Let mathematical description of nonlinear ACS is given in the form of:

$$\dot{x} = f(x),$$

where $f(0) = 0;\ t = t_0,\ x(t_0) = x_0,\ x_0 \neq 0.$

In the case of small deviations it is possible to present a nonlinear system in linear form ($u(t) = 0$) as the following:

$$\dot{x} = Ax,$$

where $x \in R^n$, $A(n \times n)$ is a matrix of coefficients;

coordinates of balance (rest) $X = \begin{vmatrix} 0 \\ 0 \\ \ldots \\ 0 \end{vmatrix}.$

If the balance point $X=0$ is stable, then around the beginning of coordinates there exists an area of attraction of trajectories, area L_0 (fig. 1).

If it is known only, that attraction area exists, it is believed, that *balance condition is stable "in small".*

Stability "in small" states the fact of presence of stability, but does not define the value of the beginning of deviations.

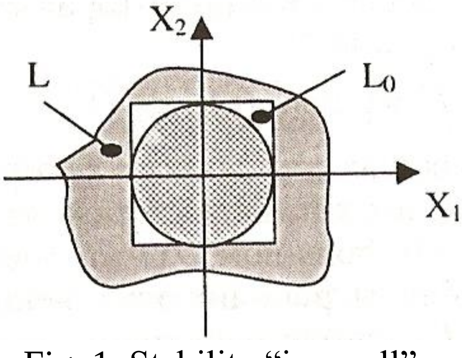

Fig. 1. Stability "in small"

Stability "in big". Let us suppose that in our system working functioning area of the system L_0 (in deviation) is known:

$$\dot{x} = f(x).$$

In case $|x| \leq L_0$ is in the process of functioning, i.e. it is working area of deviations of the system.

If area L_0 totally belongs to area L, *the balance is stable "in big"* (fig. 3). If the part of the area is outside the area L, then balance is stable *"in small",* but instable *"in big"* (fig. 2).

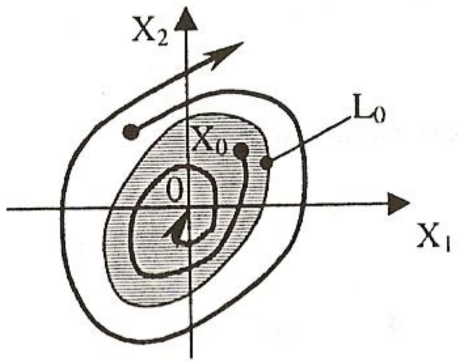

Fig. 2. Stability "in small", instability "in big"

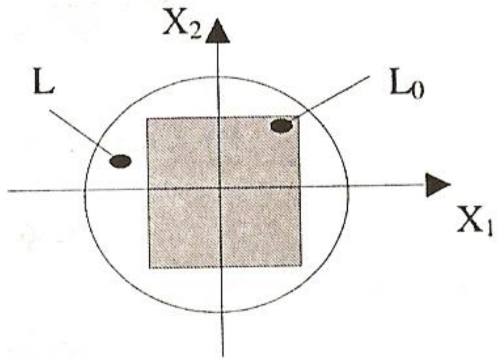

Fig. 3. Stability "in big"

Every point inside the shaded area will come to the beginning of coordinates. Let us consider an example.

Example 6.5. Let it be a nonlinear system. Working area of functioning system L_0 is known, and it contains a point $X=0$ (the beginning of coordinates):

$$L_0 = \left\{ x : \lim_{t \to 0} x(t, t_0, x_0) = 0 \right\} -$$

a set of variable conditions, in which at any value of x leads to the beginning of coordinates.

Stability "in big" will have place, if $L_0 \in L$, where L_0 is an area of working deviation of the system.

If area L spreads over the whole space, balance is called stable "as a whole" (fig. 4). If

$$\dot{x} = f(x),$$
$$f(0) = 0$$

it means that area L obligatory includes the beginning of coordinates.

Area of stability L will be the whole space of variable conditions. For linear system stability "as a whole" is shown in fig. 5.

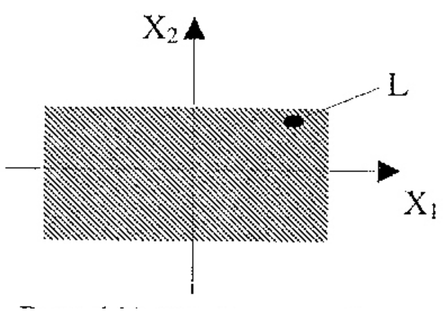

Fig. 4. Stability "as a whole"

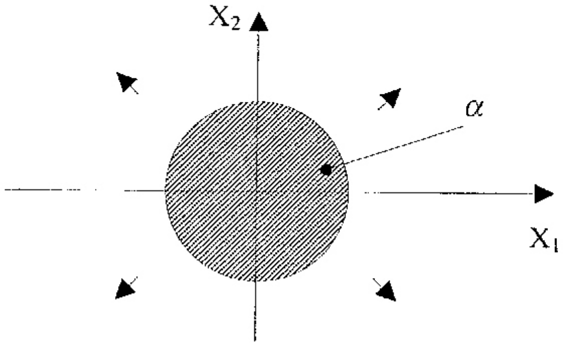

Fig. 5. Stability of a linear system

If $|x_0| = \leq \alpha$, where α = const, *the system will be stable everywhere.*

For research of stability "in big" and "as a whole" special methods are used, two of them will be presented in brief further:

– second (direct) of A.M. Lyapunov;

– method of V.M. Pópov for research absolute stability (i.e. stability "as a whole" for defined class of nonlinearities).